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STEM education in ancient Mesopotamian period: Looking through few Mathematics Problems

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Abstract: This paper attempts to see mathematical teaching situations through few selected problems of Babylonian period which were worked out by other researchers. Thus, it was possible to relate present day situations both at colleges and in the construction fields. As mathematics is basic science and Sumerian – Babylonian contributed place value number system for future development of science, the study shows a streak in the past education system.

Keywords: Babylonian, Cuneiform, Diagonals, Mathematics, Square root

Introduction

George (2005) sees that edubba literature of Sumerians reflect the educational practice of the period of the king Sulgi reigns and his successors; he quotes Sulgi writing that for all eternity the edubba was never to change and for all eternity the place of learning was never to cease functioning. He, also notes that the kind of education of those periods spread out to other crafts and became traditional, all over the world. The quality expectation in edubba were very high with constant supervision, training, overseeing, rejection of wrong or bad clay tablets and heaping these destructed pieces immediately. He notes these destructed pieces were recycled, that such clay lumps could be seen in households from the archaeological diggings. Kramer (1971) notes that the school children prepared all sorts of mathematical tables and worked on many detailed problems and solutions. They were extensively taught on writing, reading variety of literary compositions, poems, but, little is known of the teaching methods and techniques practiced in the Sumerian schools. He notes that the major scientific achievement of Sumerian as contribution to humanity is by devising sexagesimal number system with place values.

Postgate (2004) highlights a student's self-assessment statement of Sumerian school as he or she could write tables of grains measures from 1-gur to 600-gur and weight measures of 1-shekal to 20-minas of silver. He enumerates use of mathematics in those ancient times by field surveyors, architects, irrigation, temple and army accountants. Schmandt (1982) sees Sumerians developed the counting system, gradually, from concrete to abstract forms, from 8000 BC to 2500 BC, and towards what we are very familiar with modern abstract mathematics. We could expect lots of efforts in thinking, correcting, teaching and training. Purushotaman and Suresh (2014,2021) reported about the Sumerian school environment as to very similar to our recent past of traditional Gurukula and Tamilnadu thinnai-palli systems, that students were trained to sharpen their eyes and they were trained to become leaders of their times. The education quality system was very strict with punishment, close supervision and hard work. They were asked to find solutions to problems faced by people and find out answers on improving agriculture technology and to enhance water resources, that we do still face these problems even today. Further, Menon and Purushothaman(2017) brought out the possibility to refine our understanding of Sumerian literature through present form Tamil. Fowler and Robson (1998) through their research made our understanding of Babylonian mathematics better and enable us to see teaching situations of mathematics during Mesopotamian period. They note that most of the mathematical tablets were students' reference tables, exercise problems and solutions procedures; many were works of the students. In this paper we attempt to see the teaching situations of mathematics through few basic algorithms developed by Hoyrup (2002,2017) linking these at elementary level with modern thoughts (Dellajustina & Martins, 2014).

Mathemicatics

Square root of a number

Hoyrup (2002, 2017) tell us that Sumerian surveyors, engineers and architects saw their field problems as applied mathematics and saw land areas as squares, rectangles and triangles and in geometrical forms. Fowler & Robson (1998) and Hoyrup (2002, 2017) rightly explain the numerical solutions through geometrical representations. They give easy to visualize algorithms for working out diagonals, presumably of right-angle triangles, squares and rectangles. Given below is a slightly modified expression of what by Fowler and Robson (1998) had given to find out a square root of a number (say as seen in Cuneiform Tablet catalogue no. YBC 7243 as a possible solution):

Taking any number N, whose square root is to be found out with initial value of ai, the improved value of $\{a_{i+1}\}$ could be obtained as

$$\{a_{i+1}\} = a_i + [(N - a_i^2) \div 2a_i]$$

The initial value is a near guess of the square root and the difference between the given number N and square of initial guess is seen as difference in area, and factored correction is applied. Høyrup notes that Babylonians thought that side of a square is square root of the square area, and we see the correction factor maintaining dimensional equality and direction. Another equivalent expression often found, but abstract is

$$\{a_{i+1}\} = (½) \text{ x } \{a_i + (N \div a_i)\}$$

Diagonal of a triangle or a rectangle

The gate problem (as seen in Cuneiform Tablet catalogue no. VAT 6598 #6) and its solution given is interesting, particularly using of the similar expression or form used for square root but not as an iterative solution (Gordon 1912;Fowler & Robson 1998; George 2005).

The height of the gate is, H, it's base is B, but H is much higher than B, then the diagonal of the gate is given as

$$D = [H + \{½ \text{ x } (B^2 \div H)\}]$$

There is another problem (as seen in Cuneiform Tablet catalogue no. VAT 6598 #7) mentioned by Høyrup (2002) that he initially exclaims it as nonsensical. As the solution procedure given in the Babylonian tablet is interpreted, translated and read through, he gives the expression for the diagonal as,

$$D = [H + (2 \text{ x } B^2 \text{ x } H)]$$

Even we can get the doubt whether this tablet was a student's answer written in an examination. An attempt is made to revisit the transliteration as given Hoyrup (2002, 2017) but through Tamil, for the problem (VAT 6598 #7) having some likely errors; however, keeping the translation and interpretation given by Høyrup, mostly intact. This attempt is provided herein below by translating back in English:

Problem

How will you get the diagonal, if height is 40 units and bottom width is 2 units? Consider 10 as tips to solve the problem?

Solution:

1. Taking (2/12) as (1/6) as 10(in sexagesimal) multiplying by itself get 1,40 as small side; multiply that with 40 (to get 1,06,40).
   [That is B^2xH].
2. With 1, 06, 40 multiply by two to get 2, 13, 20 and to that add 40 the height.
   [That is $\{(B^2xH) \text{ x}2\} + H$]
3. we get 42, 13, 20 and say that is the diagonal. [Note as given sides are 40 and 2; the above answer of 42,13,20 in decimal system would be 42.22 which is not correct for the given side values of a triangle.
4. The correct value in decimal system would be 40.05; interestingly, if we apply the gate problem method here, that is 40+ $\{(0.5x2^2) \div 40\}$, again the diagonal would be 40.05;

It is also interesting that both methods were present in the same tablet, that it was not destroyed and thrown as debris. This requires further studies on different methods used by the Babylonians at different places and at different periods. Seeing the transliterations through Tamil was not much helpful to resolve this particular problem (VAT 6598 #7) that appear to have error.

Surveyors

Surveying has occurred since humans built the first large structures. Cooper (2013) has attempted to consolidate views on development of field surveying and office work calculations, which were then used mainly for irrigation, buildings, temples construction and commercial purpose ever since 3500BC, by Sumerians and Babylonians in the past. Cooper (2013) notes from the Sumerian literature that surveying techniques were passed down from gods to kings and the deity for surveying was a goddess. He highlights that field works were done by male and office work calculations by female scribes. He also highlights use of line threads and measuring rods and artefacts depicting such use. Further, he notes that the archaeological evidences also suggest use surveying techniques and good practices.

Cooper (2013) highlights that in Mesopotamia, the students after mastering the rudiments of metrological units and arithmetic were given numerical problems to solve. Where, the problems were relating to accounting, metrology, land surveying, construction and commerce. We find field surveying to have started during early civilizations. They started building structures such as construction of pyramids by Egyptians, Ziggurats by Sumerian, planned townships by Indus valley Civilization. Moreover Hoyrup (2002,2017) study reveals that the terms relating to surveyor and surveying have similar root words in Sumerian and Tamil. For example, fixing boundary contours in a pond using water level is called in Tamil as, "mulai maal" whereas one of a term for surveyor in Sumerian is, "mul'. A major application of mathematics during Babylonian period was also in the field of surveying. Babylonians knew how to divide trapeziums into two equal parts and many of the field problems could have come from surveyors and used to train students or apprentices. Hoyrup (2002,2017) and Postgate (2004) refer to surveyor field plan reworked out to scale by Thureau-Dangin's work (refer p-231 fig-12.3 of Postgate or p-102, fig-44 of Høyrup). As field engineers we can easily identify the field plan very similar to our closed traversing with baselines, offsets and forming rectangles, triangles and trapeziums, with all measurements, just the way we do chain survey using cross staff. We will not be drawing sketches to scale in the field, but do it at office. The correct to scale map or sketch as shown in the reference tell us our own experience how we come across with very irregular boundary in the field. Babylonians would have used cross staff to fix right angles in the field work which is similar to our current surveying methodology. Even, they would have used arc methods as they workout at office, all the three sides. Or, they could have used right set squares, even today we use wooden or steel frames to turn right angles.

Sumerian-Babylonian surveying has possibly paved way for today's surveying techniques and method. As Indus valley and Mesopotamian had trade relations; we could expect both to have taken advantage of new developments in science, engineering, technology and mathematics of their period. The Tamilnadu government rightly claims that Indian surveying was very ancient right from Indus period, Chola king Karikala period based on their works and even of modern British period in India; the first research was carried out by Captain Priestly (TNSCDA;TNSurvey2021). There were surveying manuals in use during the British period in India as per Gordon(1912) studies. Construction of dams, canals and townships were impossible without use of planning and surveying. It is also interesting to note that the British period survey manual (Gordon 1912) talks about traverse chain survey and even shows how areas of trapeziums are worked out. Postgate (2004) quotes from various studies that architects used mathematics, field lines which were straight and right angles were accurate. As George (2005) had noted on spreading of craftsmanship to other parts of the world, we could expect surveying methods to have spread out. Now this part history will have to be connected to the modern history with the British period. The history of survey of India happened in 18th century. Lt. Col. William Lambton started measuring India in 1802 under General Wellesley during a scientific survey to map and measure the Indian subcontinent land. His team, which was compiling the survey manually, acquired the name of the Great Trigonometrical Survey of India. Now a days if anybody wants to measure land can simply calculate the distance on google maps using AutoCAD application. The purpose of surveying remains constant irrespective of any civilization, the tools used to accomplish them have evolved drastically with human advancement. Revolutionary changes make Modern surveying techniques in digital manner. Equipment's like total station, GPS, Remote sensing, aerial photogrammetry, digital levels electronic theodolites has played a vital role in modern society.

Further discussions

We carried out a preliminary exploratory investigation on Sumerian terms relating to buildings and mathematics considering the transliterations, which would show terms presently used in India and in Tamilnadu. The study indicated some scope for further research, particularly Sumerian – Babylonian mathematical problems, solutions and procedures might give better opportunities to identify those languages and terms still in use. Hoyrup (2002,2017), highlighted how he could improve upon the earlier translation works with respect to understanding Babylonian mathematical problems and their solutions and how difficult was the research work George (2005) .The algorithm proposed by Fowler and Robson (1998) based on their and other's work appear to be very concrete and easy to visualize for students, particularly from field surveying specializations. However, the solutions to the gate problem (VAT 6598 #6) and a similar problem (VAT 6598 #7), but as appearing with an ambitious solution procedure are intriguing. The VAT 6598 #7 problem appear to use thumb rules say like students averse to using reciprocal tables or students under prepared or imaginative, though it is premature to label the writer of that cuneiform tablet.

Finding the diagonal problem (VAT 6598 #7) has probably mixing of units though both length units are same only base length is converted as 1/6th of a rod or kazhi (pole) but height is kept as cubits than converting into rods, this would have made a quick answer like what we use as few personal level thumb rules in the field. This also, alternatively remind us as students how we get confused while solving problems with mixing units, though dimensional homogeneity might be there. We shall also consider the high-quality expectations at Sumerian schools that tablets written with errors were immediately destroyed and sent for recycling. We learn similar quality approach was there during construction of Tanjore temple that the chief inspector would apply his hammer on imperfect stone works with defects, sending it to debris heaps. These observations suggest us to look for other similar tablets with similar solutions which could give us better picture. Of interest are both solutions for square root and finding of the diagonal as in the case of the gate problem. When it comes to solution of square roots, we cannot stop wondering how later days methods evolved with or without Babylonian Mathematics on Pell's equation and Newton – Raphson methods (Ramasamy 2004; Dellajustina et al., 2014). The Babylonian gate problem gives some indications of using half tangent value to find an approximate value of arc length to be added to the height, as engineers, many times we are satisfied with approximate values in the field. But another question arises on what was the need to find out a diagonal, if we can set a right angle with known bottom width and height, except when double checking is required or when only side slope is available than the vertical face for measurements. Naturally, the Babylonian mathematical problems, solutions procedures appear for a field survey engineer as office work and we have to look at how field work was managed in those days, though we get few indications through few poem lines how Gudea set the architect plan on the ground. Though these studies show historical development of science, technology, engineering and mathematics and how students were taught in those early development days give us confidence to progress and look ahead on future of humanity.

Mathematics has become such a specialized subject, if any Babylonian comes back, he or she would be astonished to see the current developments in the field. The study of mathematics from Babylonian period and how they used rough work tablets would be interesting and inspiring all of us.

Conclusion

A small attempt was made to understand how mathematics was taught by going through few selected problems. These problems pertaining to square roots as sides of square area or finding out of diagonals for square and rectangle areas offer interesting settings. We could relate present student and field engineers' situations as was then at Babylonian period. We also learn and appreciate development of mathematics as from concrete learning to abstract conceptualization in the course of time. There are lots of scope for research in this area with concrete mathematical problems, procedures and solutions both reading from tablets, transliteration, translation and relating to present day solutions as inspiring as any other subject of study.

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